

GGSIPO Mathematics 2005

1. The equation of the plane through the intersection of the planes $x+y+z = 1$ and $2x+3y-z+4 = 0$ and parallel to x-axis is :

- a $y - 3z + 6 = 0$ b $3y - z + 6 = 0$
c $y + 3z + 6 = 0$ d $3y - 2z + 6 = 0$

2. The distance of the point 3,8,2 from the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3}$ measured parallel to the plane $3x+2y-2z+15 = 0$ is :

- a 2 b 3
c 6 d $\frac{19}{2}$

3. Let 3,4, -1 and -1,2,3 are the end points of a diameter of sphere. Then the radius of the sphere is equal to :

- a 1 b 2 c 3 d 9

4. If A,B,C,D are the points 2,3, -1,3,5, -3,1,2,3,3,5,7 respectively, Then the angle between AB and CD is :

- a $\frac{\pi}{2}$ b $\frac{\pi}{3}$
c $\frac{\pi}{4}$ d $\frac{\pi}{6}$

5. If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is :

- a -1 b 0
c 1 d 2

6. A five digits number is formed by writing the digits 1,2,3,4,5 in a random order without repetitions. Then the probability that the number is divisible by 4, is :

- a $\frac{3}{5}$ b $\frac{18}{5}$
c $\frac{1}{5}$ d $\frac{6}{5}$

7. Two persons A and B takes turns in throwing a pair of dice. The first person to throw 9 from both dice will be awarded the price. If A throws first, then the probability that B wins the game, is :

- a $\frac{9}{17}$ b $\frac{8}{17}$

c 8/9 d 1/9

8. The probability that in year of the 22nd century chosen at random, then there will be 53 Sundays, is :

a 3/28 b 2/28

c 7/28 d 5/28

9. The standard deviation of a variable x is 10. Then the standard deviation of 50+5x is :

a 50 b 550

c 10 d 0.98

10. The octal equivalent of the decimal number 0.3125 is :

a 0.24 b 0.42

c 0.39 d 0.98

11. The hexadecimal equivalent of the binary number 111100001010001 is

a 15C3 b C351

c 3C51 d C315

12. A real value of x will satisfy the equation $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta$ α and β are real, if :

a $\alpha^2 - \beta^2 = -1$ b $\alpha^2 - \beta^2 = 1$

c $\alpha^2 + \beta^2 = 1$ d $\alpha^2 - \beta^2 = 2$

13. If ω is a complex cube root of unity, then the value of

$\frac{p+q\omega+r\omega^2}{r+p\omega+q\omega^2} + \frac{p+q\omega+r\omega^2}{q+r\omega+p\omega^2}$ $p, q, r \in \mathbb{R}$ is equal to :

a 0 b 1

c -1 d 2

14. If P, Q, R, S are represented by the complex numbers $4 + i, 1 + 6i, -4 + 3i, -1 - 2i$ respectively, then PQRS is a :

a rectangle b square

c rhombus d parallelogram

15. If n is a positive integer, then $1+i^n + 1-i^n$ is equal to :

a $\sqrt{2}^{n-2} \cos\left(\frac{n\pi}{4}\right)$

b $\sqrt{2}^{n-2} \sin\left(\frac{n\pi}{4}\right)$

c $\sqrt{2}^{n+2} \cos\left(\frac{n\pi}{4}\right)$

d $\sqrt{2}^{n+2} \sin\left(\frac{n\pi}{4}\right)$

16. The number of ways in which 9 persons can be divided into three equal groups is :

a 1680 b 840

c 560 d 280

17. A dictionary is printed consisting of 7 letters words only that can be made with a letters of the word CRICKET.If the words are printed are alphabetical order is an ordinary dictionary,then the number of words are before the word CRICKET is :

a 530 b 480

c 531 d 481

18. If the sum of the coefficient in the expansion of $x+y^n$ is 1024, then the value of the greatest coefficient in the expansion is :

a 356 b 252

c 210 d 120

19. The value of the determinant

$$\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix} \text{ is :}$$

a $2 \cdot 10! \cdot 11!$ b $2 \cdot 10! \cdot 13!$

c $2 \cdot 10! \cdot 11! \cdot 12!$ d $2 \cdot 11! \cdot 12! \cdot 13!$

20. If A and B are 3x3 matrices such that $AB = B$ and $BA = A$,than :

a $A^2 = A$ and $B^2 \neq B$

b $A^2 \neq A$ and $B^2 = B$

c $A^2 = A$ and $B^2 = B$

d $A^2 \neq A$ and $B^2 \neq B$

21. If the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are collinear, then the rank of the matrix

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \text{ will always be less than :}$$

- a 2 b 3
c 1 d none of these

22. The system of equations; $x+y+z = 6, x+2y+3z = 10, x+2y+\lambda z = 6$ has number solution for :

- a $\lambda = 3, \mu = 10$ b $\lambda = 3, \mu \neq 10$
c $\lambda \neq 3, \mu \neq 10$ d none of these

23. If $A = \begin{bmatrix} \sin(\theta + \alpha) & \cos(\theta + \alpha) & 1 \\ \sin(\theta + \beta) & \cos(\theta + \beta) & 1 \\ \sin(\theta + \gamma) & \cos(\theta + \gamma) & 1 \end{bmatrix}$ Then :

- a $A=0$ for all θ
b A is a odd function of θ
c $A=0$ for $\theta=\alpha+\beta+\gamma$
d A is a independent of θ

24. An investigator interviewed 100 students to determine the performance of three drinks milk, coffee and tea; 20 students take milk and coffee, 30 students take coffee and tea, 25 students take milk and tea, 12 students take milk only, 5 students take coffee only and 8 students take tea only. Then the number of students who did not take any drinks any of three, is :

- a 10 b 20
c 25 d 30

25. Let $Y = \{1, 2, 3, 4, 5\}$, $A = \{1, 2\}$, $B = \{3, 4, 5\}$ and ϕ denotes null set. If $A \times B$ denotes Cartesian product of the sets A and B, then $Y \times A \cap Y \times B$ is :

- a Y b A
b B d ϕ

26. Let $A = \{2, 3, 4, 5, \dots, 16, 17, 18\}$. Let \approx be the equivalence relation on $A \times A$ Cartesian product of A and A, defined by $a, b \approx c, d$ if $ad = bc$, then the number of ordered pairs of the equivalence class of 3, 2 is :

a 4 b 5 c 6 d 7

27. A question 'who have studied Physics?' was asked to three students A,B and C. The question was answered correctly as it is true that If A studied Physics, then B also studied Physics but it is false statement that if C studied Physics, then B also studied physics. Then physics was studied by :

a both A and B b only A
c only B d only C

28. If a,b be two fixed positive integers such that $f(a+x) = b + [b^3 + 1 - 3b^2 f x + 3b\{f x\} - \{f(x)^3\}^{1/3}]$ for all real x, then f(x) is a periodic function with period :

a a b 2a
c | b d 2b

29. The domain of the function $f(x) = \log_3 + x^2 - 1$ is :

a $-3, -1 \cup 1, \infty$
b $[-3, -1] \cup [1, \infty$
c $-3, -2 \cup -2, -1 \cup 1, \infty$
d $[-3, -2 \cup -2, -1 \cup 1, \infty$

30. The value of $\cot 70^\circ + 4 \cos 70^\circ$ is :

a $1/\sqrt{3}$ b $\sqrt{3}$
c $2\sqrt{3}$ d $\frac{1}{2}$

31. The equation of $\sin x + \sin y + \sin z = -3$ for $0 \leq x \leq 2\pi, 0 \leq y \leq 2\pi, 0 \leq z \leq 2\pi$ has :

a one solution
b two sets of solution
c four sets of solution
d no solution

32. If $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x, x \geq 0$ then the smallest interval in which θ lies is :

a $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4}$ b $0 \leq \theta \leq \frac{\pi}{4}$
c $-\frac{\pi}{4} \leq \theta \leq 0$ d $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

33. Let A,B and C are the angles Of a plain triangle and $\tan\left(\frac{A}{2}\right) = \frac{1}{3}, \tan\left(\frac{B}{2}\right) = \frac{2}{3}$. Than $\tan\left(\frac{C}{2}\right)$ is equal to :

- a 7/9 b 2/9 c 1/3 d 2/3 2/3

34. If α, β $\alpha \neq \beta$ satisfies the question $a \cos \theta + b \sin \theta = c$, then the value of $\tan\left(\frac{\alpha+\beta}{2}\right)$ is :

- a b/a b c/a c a/b d c/b

35. A ray of light passing through the point 1,2 is reflected on the x -axis at a point P and passes through the point 5,3, then the abscissa of a point P is :

- a 3 b 13/3
c 13/ 5 d 13/4

36. The equation $4x^2 - 24xy + 11y^2 = 0$ represents :

- a two parallel lines
b two perpendicular lines
c two lines through the origin
d a circle

37. The length of the chord joining the points in which the straight line $\frac{x}{3} + \frac{y}{4} = 1$ cuts the circle $x^2 + y^2 = \frac{169}{25}$ is :

- a 1 b 2
c 4 d 8

38. The normal to the parabola $y^2 = 8x$ at the point 2,4 meets the parabola against the point :

- a -18,-12 b -18,12
c 18,12 d 18, -12

39. If a bar of given length moves with its extremities on two fixed straight lines at right angles, then the locus of any point on bar marked on the bar describes a/an :

- a circle b parabola
c ellipse d hyperbola

40. The straight line $x+y=\sqrt{2}p$ will touch the hyperbola $4x^2-9y^2=36$ if :

- a $p^2 = 2$ b $p^2 = 5$

c $5p^2 = 2$ d $2p^2 = 5$

41. The function $f(x) = \frac{1 - \sin x + \cos x}{1 + \sin x + \cos x}$ is not defined at $x = \pi$. The value of $f(\pi)$, so that $f(x)$ is continuous at $x = \pi$, is :

- a $-1/2$ b $1/2$
c -1 d 1

42. If $f(x) = \sin^2 x$ and the composite function $gf(x) = |\sin x|$, then the function $g(x)$ is equal to :

- a $\sqrt{x-1}$ b \sqrt{x}
c $\sqrt{x+1}$ d $-\sqrt{x}$

43. Area of the figure bounded by the curves $y = |x-1|$ and $y = 3 - |x|$ is :

- a 1 sq. units b 2 sq. units
c 3 sq. units d 4 sq. units

44. Let $x = \left[\frac{a+2b}{a+b} \right]$ and $y = \frac{a}{b}$, where a and b are positive integers. If $y^2 > 2$, then :

- a $x^2 \leq 2$ b $x^2 < 2$
c $x^2 > 2$ d $x^2 \geq 2$

45. $\int_0^1 \tan^{-1} \left(\frac{1}{x-x+1} \right) dx$ is :

- a $\log 2$ b $-\log 2$
c $\frac{\pi}{2} + \log 2$ d $\frac{\pi}{2} - \log 2$

46. The curves $x = \log y + e$ and $y = \log \left(\frac{1}{x} \right)$:

- a do not meet
b meet at one point
c meet at two points
d meet at more than two points

47. $\lim_{x \rightarrow 0} \frac{\cos(\sin x) - 1}{x^2}$ equals :

- a 0 b -1

c $\frac{1}{2}$ d $-\frac{1}{2}$

48. Let $\vec{i}, \vec{j}, \vec{k}$ be three vectors from $\vec{i} \times \vec{j} \times \vec{k} = \vec{i} \times \vec{j} \times \vec{k}$ it :

a $\vec{i} \times \vec{j} \times \vec{k} = 0$ b $\vec{i} \times (\vec{j} \times \vec{k}) = 0$

c $\vec{i} \times \vec{j} = \vec{i} \times \vec{k}$ d $\vec{i} \times \vec{j} = \vec{j} \times \vec{k}$

49. If $\vec{i}, \vec{j}, \vec{k}$ are units vectors and $|\vec{a}| = a$, then the value of

$|\vec{i} \times \vec{a}|^2 + |\vec{j} \times \vec{a}|^2 + |\vec{k} \times \vec{a}|^2$ is :

a a^2 b $3a^2$ c $2a^2$ d $4a^2$

50. If the area above the x-axis bounded by the curves $y = 2^{kx}$ and $x = 0$ and $x = 2$ is $\frac{3}{\log 2}$, then the value of k is :

a $\frac{1}{2}$ b 1 c -1 d 2

51. The value of $\int_a^b \frac{x}{|x|} dx, a < b < 0$ is :

a $-|a| + |b|$ b $|b| - |a|$

c $|a| - |b|$ d $|a| + |b|$

52. The value of

$\int_{-2}^2 \left[p \log \left(\frac{1+x}{1-x} \right) + q \log \left(\frac{1-x}{1+x} \right)^{-2} + r \right] dx$ depends on:

a The value of p

b The value of q

c The value of r

d The value of p and q

53. A curve having the condition that the slope of tangent at some point is two times the slope of the straight line joining the same point to the origin of co-ordinates, is a/an :

a circle b ellipse

c parabola d hyperbola

54. If a is an arbitrary constant, then solution of differential equation

$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is :

- a $x\sqrt{1-y^2} + y\sqrt{1-x^2} = a$
- b $y\sqrt{1-y^2} + x\sqrt{1-x^2} = a$
- c $x\sqrt{1-y^2} - y\sqrt{1-x^2} = a$
- d $y\sqrt{1-y^2} - x\sqrt{1-x^2} = a$

55. A particle is moving along the curve $x = at^2 + bt + c$. If $a = b^2$, then the particle would be moving with uniform :

- a rotation b velocity
- c acceleration d retardation

56. The solution of the equation $x^2 \frac{d^2y}{dx^2} = \log x$ when $x=1, y=0$ and $\frac{dy}{dx} = -1$ is :

- a $\frac{1}{2} \log x^2 + \log x$
- b $\frac{1}{2} (\log x^2 - \log x)$
- c $-\frac{1}{2} \log x^2 + \log x$
- d $-\frac{1}{2} \log x^2 - \log x$

57. Let the unit vectors \vec{i} and \vec{j} be perpendicular to each other and the unit vector \vec{r} be inclined at an angle θ to both \vec{i} and \vec{j} . If $\vec{r} = \alpha \vec{i} + \beta \vec{j} + \gamma \vec{k}$, where α, β, γ are scalars, then :

- a $\alpha = \cot \theta, \beta = \sin \theta, \gamma^2 = \cos 2\theta$
- b $\alpha = \cos \theta, \beta = \cos \theta, \gamma^2 = \cos 2\theta$
- c $\alpha = \cos \theta, \beta = \sin \theta, \gamma^2 = \cos 2\theta$
- d $\alpha = \sin \theta, \beta = \cos \theta, \gamma^2 = -\cos 2\theta$

58. If $y = \frac{1}{\sqrt{a^2-b^2}} \cos^{-1} \left[\frac{a \cos(x-a)+b}{\rho} \right]$ where $\theta = a+b \cos x - \alpha$, then $\frac{dy}{dx}$ is equal to :

- a $1/\theta$ b $2/\theta$
- c $1/\theta^2$ d $2/\theta^2$

59. Let K be a set of real number and $f:K \rightarrow R$ such that for all x any y $|f(x) - f(y)| \leq |x-y|^5$. If $f(3) = 7$, then the value of f(9) is equal to

- a 5 b 7
- c 9 d 11

60. If $f(x) = \frac{1}{1-x}$ then the derivative of the composite function $f[f\{f(x)\}]$ is equal to :

a 0 b $\frac{1}{2}$

c 1 d 2